# Physics 120 - Prof. David Kleinfeld - 2018
Notes on n-channel JFETs in the Active Region
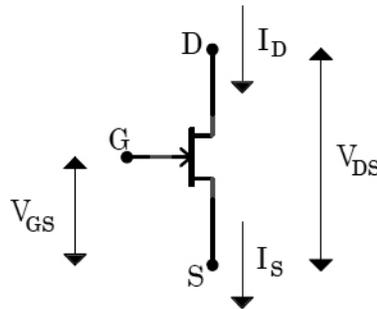

1. Basics

1. $I_G = 0$
2. $I_D = I_S \equiv I_{DS}$

In active region, device characteristics are defined by[1]:

1. $V_{GS}(\text{off}) \le V_{GS} \le 0$
2. $V_{DS} > V_{GS} - V_{GS}(\text{off})$; recall that both V_{GS} and $V_{GS}(\text{off})$ are negative
3. I_{DS} is independent of V_{DS} (ideal current source)

4. I_D is function of V_{GS}, with $I_{DS} = \dfrac{I_{DSS}}{V_{GS}^2(\text{off})}\left[V_{GS} - V_{GS}(\text{off})\right]^2 = I_{DSS}\left[1 - \dfrac{V_{GS}}{V_{GS}(\text{off})}\right]^2$

This implies $I_{DS} = I_{DSS}$ for $V_{GS} = 0$.

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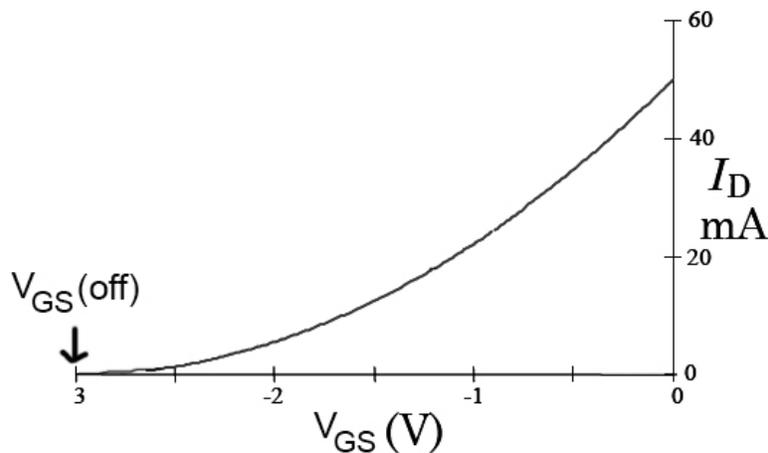
[1] The turn-off gate-to-source voltage $V_{GS}(\text{off})$ has a number of aliases, such as threshold voltage or pinch-off voltage, denoted $V_{GS}(\text{off}) = V_T = V_P = V_{PO} = V_{P0}$ The active region is also called the "saturation region" or "pentode region", while the Ohmic region is also called the "linear region" or the "triode region".

I_D – Drain Current (mA) vs *V_{DS} – Drain-Source Voltage (V)*, with $V_{GS(off)} = -3\,V$ and curves for $V_{GS} = 0\,V$, $-0.5\,V$, $-1.0\,V$, $-1.5\,V$, $-2.0\,V$, $-2.5\,V$.
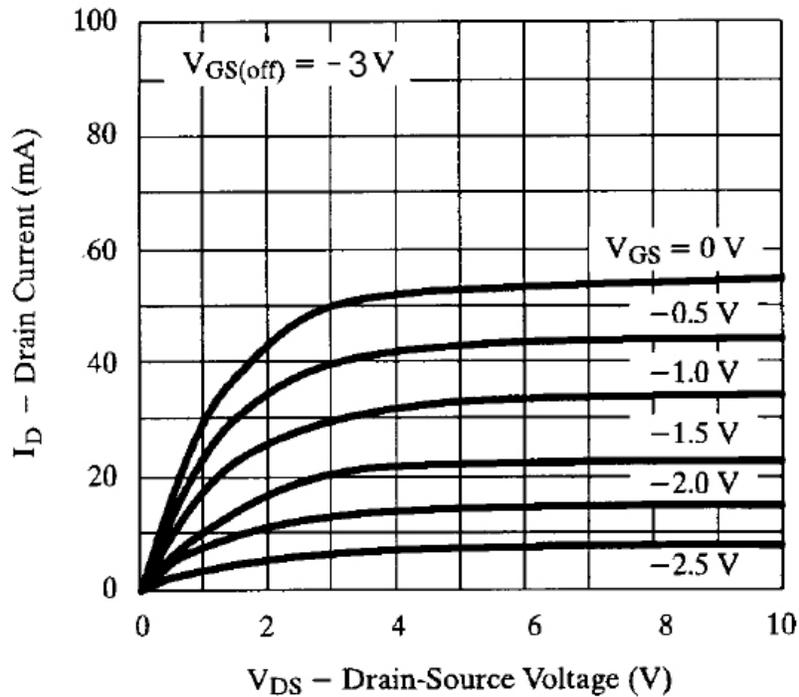
**Transconductance: The small signal limit**

For small changes in gate voltage, we can calculate the changes in source or drain current. The constant of proportionality is referred to as the transconductance, denoted g_m, where

$$g_m = \frac{dI_{DS}}{dV_{GS}} = 2\frac{I_{DSS}}{V_{GS}(\text{off})}\left[1 - \frac{V_{GS}}{V_{GS}(\text{off})}\right]$$
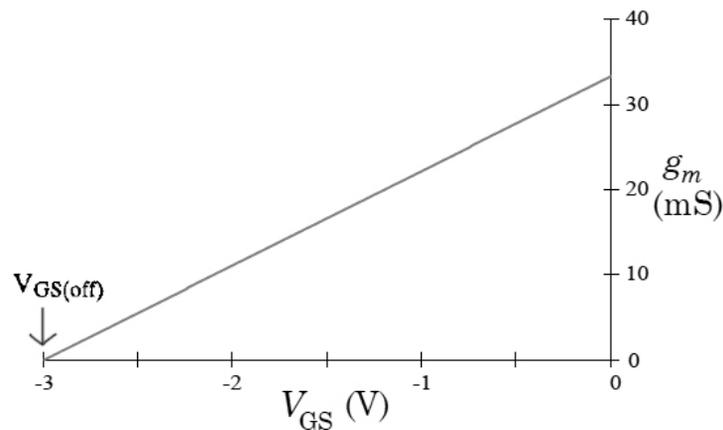
so that $\Delta I_S = g_m\, \Delta V_{GS}$. We will see later that the transconductance plays a role analogous to β with bipolar junction transistors, but is *not* a constant, *i.e.*, it depends of V_{DS}!

2. Fixed current source

Let's now consider the world's simplest current source.
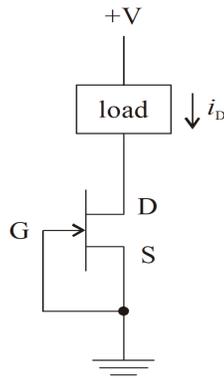
Here $V_{GS} = 0$, so the current is forced to be I_{DSS}. This current is maintained so long as the load line can accommodate the condition to maintain a value of V_{DS} in the active region, i.e., $V_{DS} > V_{GS} - V_{GS}(off)$, which reduces to $V_{DS} > - V_{GS}(off)$.

For example, the 2N5485 has $I_{DSS} = 8$ mA, enough to drive a typical LED.

The load line is given by writing Kirchoff's rule for voltage drops and ignoring the transconductance $1/g_m$:

$0 = -V_{DD} + I_{DS} R_{Load} + V_{DS}$

to yield the load line:

$$I_{DS} = \frac{V_{DD} - V_{DS}}{R_{Load}}$$

This must intercept $I_D = I_{DSS}$ in the active region. Noting that $V_{DS} > - V_{GS}(off)$, this implies

$$R_{Load} = \frac{V_{DD} - V_{DS}}{I_{DSS}} < \frac{V_{DD} + V_{GS}(off)}{I_{DSS}}$$

Here, we slide along the (flat) line of $I_{DS} = I_{DSS}$ so long as $V_{DS} > - V_{GS}(off)$.

This source suffers from having a value of I_D that is not adjustable and that may vary with manufacturing!

3. Improved current source

A more sophisticated source uses a resistor between the source and ground to determine I_{DS}.
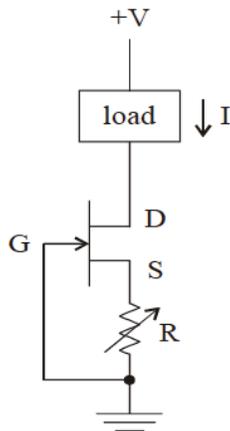
Here we have $V_G = 0$ since the gate is grounded but $V_{GS} < 0$. The loop equation encompassing the gate and source satisfies (ignoring the transconductance term $1/g_m$) is:

$0 = -V_G + V_{GS} + I_{DS} R_S$

or

$$I_{DS} = -\frac{V_{GS}}{R_S} .$$

We need to choose a value of R_s to fix I_{DS}. The second equation that relates I_{DS} and V_{GS} is the constitutive equation

$$I_{DS} = I_{DSS}\left(1 - \frac{V_{GS}}{V_{GS}(\text{off})}\right)^2 .$$

We are free to pick a desired set-point, or co called "quiescent current", denoted $I_{DS,Q}$, with $I_{DS,Q} < I_{DSS}$. Then the required value of R_S is found by substituting $V_{GS} = -I_{DS,Q} R_S$ into the constitutive equation, i.e.,

$$I_{DS,Q} = I_{DSS}\left(1 + \frac{I_{DS,Q} R_S}{V_{GS}(\text{off})}\right)^2 .$$
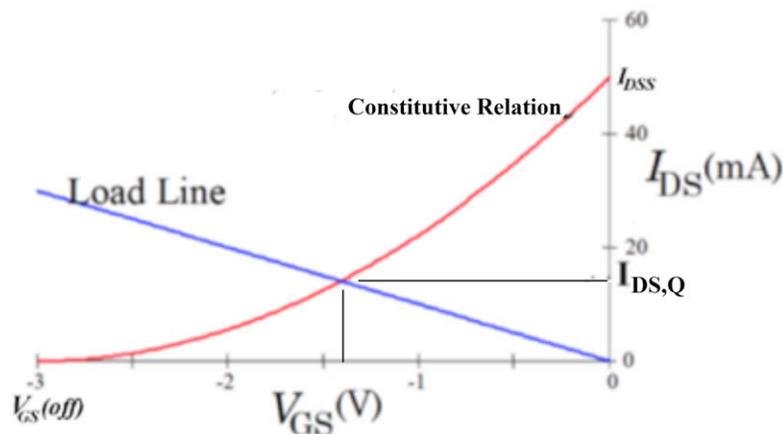
Thus

$$R_S = \frac{-V_{GS}(\text{off})}{I_{DS,Q}}\left(1 - \sqrt{\frac{I_{DS,Q}}{I_{DSS}}}\right) .$$

As an example relevant to the laboratory 7 exercise with a 2N5485, for the choice $I_{DS,Q}$ = 0.4 mA with I_{DSS} = 8 mA and $V_{GS}(\text{off})$ = - 3 V, we find R_S = 5.8 kΩ. We use the closest value 5 % resistor at 5.6 kΩ. These current sources are independent of fluctuations in the power supply voltage and largely independent of g_m.

The load line for I_D versus V_{DS} is found by computing the voltage drops along the loop, *i.e.*,

$0 = -V_{DD} + I_{DS} R_L + V_{DS} + I_{DS} R_S.$

Thus $I_{DS} = \frac{V_{DD} - V_{DS}}{R_S + R_L}$ and we slide along a curve of constant I_{DS} until we can no longer maintain

a value of V_{DS} in the active region, i.e., $V_{DS} > V_{GS} - V_{GS}(\text{off})$. As before, this limits the maximum value of R_L, since

$$R_S + R_L = \frac{V_{DD} - V_{DS}}{I_{DS,Q}} < \frac{V_{DD} - \left[V_{GS} - V_{GS}(\text{off})\right]}{I_{DSQ}}$$

or

$$R_S + R_L < \frac{V_{DD} - V_{GS} + V_{GS}(\text{off})}{I_{DS,Q}} = \frac{V_{DD} + I_{DS,Q}R_S + V_{GS}(\text{off})}{I_{DS,Q}} = \frac{V_{DD} + V_{GS}(\text{off})}{I_{DS,Q}} + R_S$$

so that

$$R_L < \frac{V_{DD} + V_{GS}(\text{off})}{I_{DS,Q}}$$

as for the simple current source. We slide along the flat line of $I_{DS,Q}$ so long as $V_{DS} > - V_{GS}(\text{off})$.

4. Voltage follower
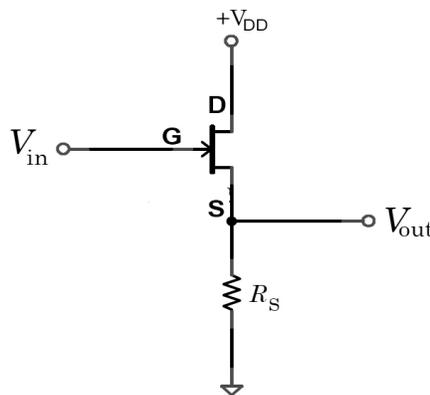
The analysis is over two loops, one to define R_S and the other to define the load line.

We consider the lower loop to relate V_{in} and I_{DS}:

$0 = - V_{in} + V_{GS} + R_S I_{DS}$

We get $V_{out} = I_{DS} R_S = V_{in} - V_{GS}$.

We see that the output follows the input with the addition of an offset term V_{GS}. Recall that $V_{GS} < 0$ so the offset is positive, but unfortunately not a constant

We can set $I_{DS} = \dfrac{V_{in} - V_{GS}}{R_S}$ equal to the constitutive relation to find an expression for V_{GS}. The

result is $V_{GS} = V_{GS}(\text{off}) \left[1 - \dfrac{V_{GS}(\text{off})}{2R_S I_{DS,Q}} \left[1 + \sqrt{1 + 4\dfrac{R_S I_{DS,Q}}{V_{GS}(\text{off})} \left(\dfrac{V_{in}}{V_{GS}(\text{off})} - 1 \right)} \right] \right]$.

Simply, V_{GS} changes substantially only for changes $\Delta V_{in} \ll V_{GS}(\text{off})$.

We consider the upper loop to relate V_{out} and I_{DS} and solve for the load line.

$0 = - V_{DD} + V_{DS} + R_S I_{DS}$

so $0 = - V_{DD} + V_{DS} + R_S I_{DS}$

and so $I_{DS} = \dfrac{V_{DD} - V_{DS}}{R_S} = \dfrac{V_{out}}{R_S}$.


Inclusion of a nonzero value of $1/g_m$:

$0 = - V_{in} + V_{GS} + (1/g_m) I_{DS} + R_S I_{DS}$

so $I_{DS} = \dfrac{V_{in} - V_{GS}}{R_S + 1/g_m}$

and $V_{out} = I_{DS} R_S = \dfrac{g_m R_S}{1 + g_m R_S}\left(V_{in} - V_{GS}\right)$.

The key is to maintain $R_s g_m \gg 1$ or $R_s \ll 1/g_m$. This is critical, as g_m is also a function of V_{GS}.

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4. Improved voltage follower

An improved follower may be built in which the offset voltage V_{GS} is minimized. We use a current source to define the current through R_1 and R_2, as show below.
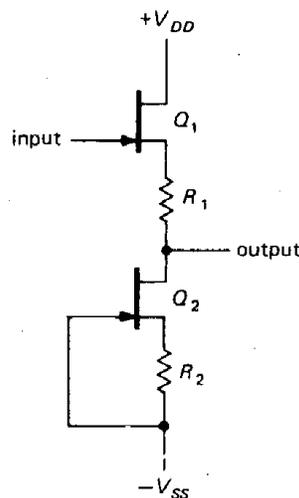
Here we may write an expression for the equilibrium current (ignoring g_m):

$0 = - V_{in} + V_{GS} (Q1) + I_{DS,Q} R_1 + V_{out}$.

But we previously solved for the self limiting current source corresponding to the lower JFET,

or $I_{DS,Q} = - \dfrac{V_{GS}\left(Q_2\right)}{R_2}$.

Then

$0 = - V_{in} + V_{GS} (Q1) - \dfrac{V_{GS}\left(Q_2\right)}{R_2} R_1 + V_{out}$

For $R_1 = R_2$ and matched JFETs (they are manufactured as pairs on a single substrate for this purpose), the output voltage is exactly the input voltage and we have a perfect follower with a very large input impedance.